

Numerical Analysis of Shear Critical RC Beams Strengthened in Shear with FRP Sheets

Denise Ferreira¹; Eva Oller²; Antonio Marí, M.ASCE³; and Jesús Bairán⁴

Abstract: The objective of this paper is to contribute to the understanding of the shear resisting mechanisms in RC beams shear-strengthened by externally bonded fiber-reinforced polymer (FRP) sheets. For this purpose, a fiber beam model of RC frames subjected to combined normal and shear forces, previously developed by the authors, has been extended to include the response of externally bonded FRP shear reinforcement in a wrapped configuration. No FRP delamination phenomena or tensile strength reductions in the corner zones are taken into account in the model. The numerical results have been compared with eight existing experimental results and the influence of the FRP sheets on the shear strength of the beam has been studied. The effects of the contribution of FRP ratio on the concrete, on the transversal steel strains and stresses, on the longitudinal tensile steel stresses, and on the diagonal compression struts have been analyzed. It is concluded that the presence of FRP reinforcement modifies the inclinations of cracks and struts, the concrete confinement stresses, and other parameters related to the shear response, producing an interaction between the concrete, internal steel, and FRP components of the shear strength. DOI: [10.1061/\(ASCE\)CC.1943-5614.0000434](https://doi.org/10.1061/(ASCE)CC.1943-5614.0000434). © 2013 American Society of Civil Engineers.

Author keywords: Reinforced concrete; Strengthening; Fiber reinforced polymer; Shear; Nonlinear analysis; Fiber beam model; Axial force-shear-bending coupling.

Introduction

The use of fiber reinforced polymer (FRP) sheets as externally bonded reinforcement (EBR) for existing RC structures has been proven to be an effective technique to lengthen the service life of structures. However, its application in the shear strengthening field is less wide because of the complexity of the resisting mechanisms that are still not well known. Despite the increasing number of existing studies of beams strengthened in shear with FRP sheets since the early 1990s (Al-Sulaimani et al. 1994; Chajes et al. 1995; Triantafillou 1998; Khalifa and Nanni 2000; Czaderski and Motavalli 2004; Deniaud and Cheng 2004; Carolin and Täljsten 2005; Boussselham and Chaallal 2006), the work is much more limited than that on flexural strengthening. In addition, a considerable scatter has been found when comparing the experimental shear resistance provided by FRP sheets, and the values predicted by codes and guidelines makes it clear that major aspects related to shear strengthening with FRP sheets are still not summarized by theoretical predictions (Boussselham and Chaallal 2008).

Therefore, there is a need to clarify the shear resisting mechanisms in FRP shear strengthened beams to design efficient solutions. After calibration, numerical models may provide information in a simpler and more economical way than conducting elaborate laboratory tests. Finite element (FE) studies on FRP shear strengthened beams were performed by Arduini et al. (1997), Malek and Saadatmanesh (1998), Vecchio and Bucci (1999), Wong and Vecchio (2003), Godat et al. (2007), and Lee et al. (2008). Most of these were two-dimensional (2D) or three-dimensional (3D) models that addressed the bonding mechanism and debonding phenomena. Despite the relative complexity of these models, they do not always provide clear or completely sustained predictions. Aspects such as strain development in the stirrups and FRP, cracking angles, and principal directions of the compression struts are not discussed in these studies. In addition, these nonlinear 2D/3D FE models are usually very complex, time consuming, and demanding. Numerical models able to predict the responses of FRP shear strengthened elements in an accurate and simple manner are needed for a wider and more efficient application of this measure in practice.

In fiber beam models, the Navier-Bernoulli plane section theory and the use of a fiber sectional discretization, combined with adequate uniaxial constitutive laws, allows fairly accurate analyses of flexural dominant RC frames to be obtained (Marí 2000). The inherent simplicity and computational efficiency make them a particularly advantageous solution for analyzing strengthened structures.

When tangential forces are applied, the plane section hypothesis is no longer valid because of the appearance of distortion. In fact, the actual distribution of shear strains and stresses in the cross section is complex and state-dependent, showing important variations while cracking is developing and approaching failure. In attempting to extend frame models to loading conditions that include shear effects, several theories for the sectional response under tangential and normal forces were developed with different levels of complexity and computational demands (Bairán and Marí 2007b). In relation to the models that use simplified approaches to model

¹Researcher, Construction Engineering Dept., Civil Engineering School, Universitat Politècnica de Catalunya, Jordi Girona 1-3, C-1 201, 08034 Barcelona, Spain. E-mail: denise.carina.santos@upc.edu

²Lecturer Professor, Construction Engineering Dept., Civil Engineering School, Universitat Politècnica de Catalunya, Jordi Girona 1-3, C-1 201, 08034 Barcelona, Spain (corresponding author). E-mail: eva.oller@upc.edu

³Professor, Construction Engineering Dept., Civil Engineering School, Universitat Politècnica de Catalunya, Jordi Girona 1-3, C-1 201, 08034 Barcelona, Spain. E-mail: antonio.mari@upc.edu

⁴Associate Professor, Construction Engineering Dept., Civil Engineering School, Universitat Politècnica de Catalunya, Jordi Girona 1-3, C-1 201, 08034 Barcelona, Spain. E-mail: jesus.miguel.bairan@upc.edu

Note. This manuscript was submitted on February 18, 2013; approved on August 8, 2013; published online on August 10, 2013. Discussion period open until February 23, 2014; separate discussions must be submitted for individual papers. This paper is part of the *Journal of Composites for Construction*, © ASCE, ISSN 1090-0268/04013016(11)/\$25.00.

shear effects, the two most widely used are the constant shear strain and the constant shear stress models. The constant shear strain assumption, because of its computational robustness, has been extensively used (Petrangeli et al. 1999; Ceresa et al. 2009). However, inaccurate results in beams highly reinforced for shear have been reported in the works of Vecchio and Collins (1988) and Mohr et al. (2010) because strains and stresses in the stirrups are deficiently simulated through this assumption. Therefore, the assumption of constant shear stress distribution was used as the basis of the shear model described in this paper. A similar fiber beam model, which includes shear effects, has been presented and validated elsewhere for the cases of RC elements without FRP strengthening (Ferreira 2013; Ferreira et al. 2013).

Plasticity based formulations allow the determination of the ultimate load carrying capacity of RC elements shear strengthened by means of FRP sheets (Caiazza et al. 2006). However, modeling the nonlinear response of FRP strengthened beams critical to shear by means of fiber beam models has not been performed. Hence, in this paper, the fiber beam model developed by the authors for the nonlinear analysis of strengthened elements, including the effects of shear, is used to predict the response of RC beams shear strengthened with FRP sheets. The advantages of this proposal are inherent, with the simplicity and straightforwardness of the fiber beam models to be applied in practical engineering analyses. The model is validated through the analysis of an existing experimental program of RC beams strengthened in shear by FRP sheets performed by Alzate (2012). In the actual format, the model does not account for delamination between FRP and concrete; hence, its range of applicability is restrained to FRP strengthening with mechanical anchorage. In this stage, only a wrapped configuration [least vulnerable to debonding, according to Teng et al. (2002)] has been considered to avoid the effects of a premature debonding failure. In further studies, a proper model for the interface bond behavior and a debonding failure criterion will be implemented to simulate other externally bonded shear strengthening configurations, such as side-bonded or U-shaped.

Description of the Model

Overview

A nonlinear fiber beam model is presented for the simulation of shear critical strengthened RC elements. Based on a flexural fiber beam model (Marí 2000), the proposed formulation is devised for 2D frame RC structures, including the interaction between axial force, shear force, and bending moment. By these means, shear effects and their interaction with normal forces are considered in both service and ultimate levels by means of a shear sensitive sectional model. This formulation is based on a hybrid (kinematic/force) approach and takes into account the multiaxial stresses generated in each fiber in addition to the nonlinearities brought by cracking and the anisotropic behavior of concrete. The element formulation is based on the Timoshenko theory and the smeared and rotating crack approach is assumed to describe the behavior of cracked concrete. Pertaining to the simulation strategy and according to Fig. 1, the concrete cross sections are discretized into fibers; the longitudinal reinforcement is simulated by means of longitudinal filaments; and transversal reinforcement, which encloses steel stirrups and external FRP, is considered to be smeared in the concrete fibers. The presented numerical model is inserted into a time-dependent and evolutive construction framework, whose detailed formulation and validation can be found in the studies by Ferreira (2013) and Ferreira et al. (2013).

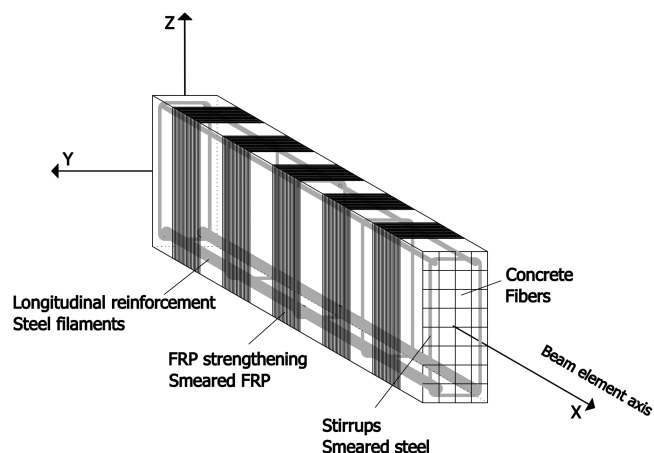


Fig. 1. Fiber beam model with M-N-V interaction for RC elements strengthened to shear with FRP

Constitutive Models

Cracked concrete is simulated as a continuous material with orthotropic nonlinear uniaxial equivalent characteristics considering full rotation of the cracks. For each stress level, the principal angle of strains, θ , is assumed to be coincident with the principal angle of stresses [Fig. 2(a)].

The stresses in each principal direction are determined accordingly to the assumed strain-stress curves and the 2D strain-stress state is given as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = D_{12} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \quad (1a)$$

$$D_{12} = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & G_{12} \end{pmatrix} \quad (1b)$$

where D_{12} = material stiffness matrix, constituted by the stiffness modulus determined in each principal direction through the constitutive laws (E_1 , E_2); and G_{12} = shear modulus determined by the expression that follows [which imposes that the angles of the principal directions of the stresses and strains are the same (Bazant 1983)]:

$$G_{12} = \frac{\sigma_1 - \sigma_2}{2(\varepsilon_1 - \varepsilon_2)} \quad (2)$$

The backbone equation used for concrete in compression is the Hognestad parabola. Depending on the biaxial stress state applied on the material, strength worsening or enhancement factors adequate the one-dimensional (1D) compressive strength of concrete to the multiaxial states. When FRP strengthening is placed by means of a wrapped configuration, the augmentation of the peak strength and ultimate strain of concrete caused by the confinement action is considered by means of the proposal of Spoelstra and Monti (1999), as represented in Fig. 2(a). The stiffness modulus, E_2 , is computed as the derivation of the backbone curve for concrete in compression. The stiffness matrix acts as an approximation of the tangent elasticity modulus, allowing the nonlinear problem to be solved by means of an iterative solution scheme. Pertaining to concrete in tension, before the onset of cracking, a linear response is assumed in which f_t and ε_{cr} correspond, respectively, to the peak

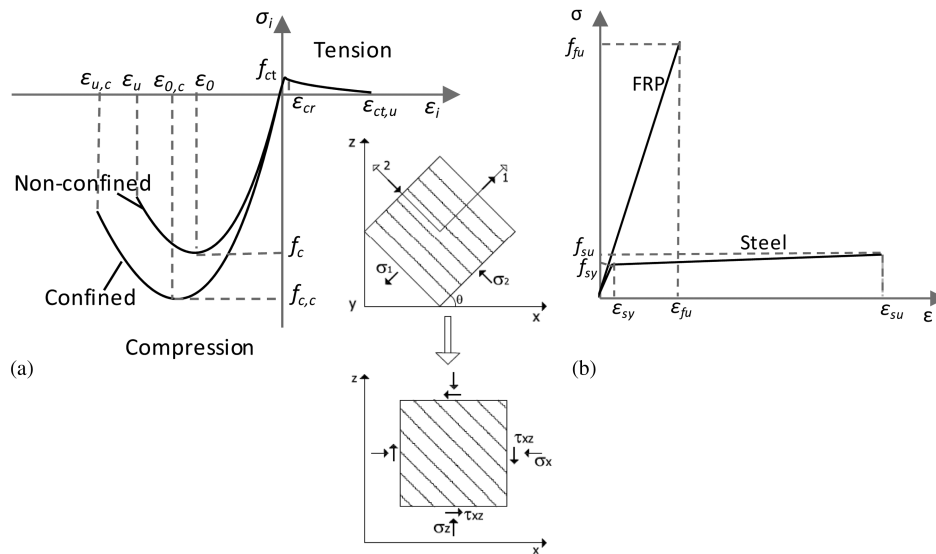


Fig. 2. Constitutive equations: (a) concrete (compression, tension, and smeared crack approach); (b) reinforcement (steel and FRP)

tensile stress and the correspondent strain. After cracking, tension softening is considered by means of the Cervenka (1985) curve, with its parameters considered as $c = \varepsilon_{sy}$ and $k_2 = 0.5$; ε_{sy} is the yielding strain of reinforcement that implies that tension stiffness is assumed to vanish after the yielding of the reinforcement. In the uncracked state, the stiffness, E_1 , is used as the elasticity modulus of concrete; after cracking, the secant stiffness is assumed.

Both steel and FRP reinforcement are simulated through a bilinear and linear 1D constitutive equations, respectively

[Fig. 2(b)]. Perfect bond between concrete and steel reinforcements is assumed.

Section Model

The key assumptions of the hybrid (kinematic/force) sectional model, including the axial force–shear–bending interaction (N-M-V), are represented in Fig. 3: (1) the kinematic assumption is the Bernoulli-Navier plane-section theory, which is coupled with

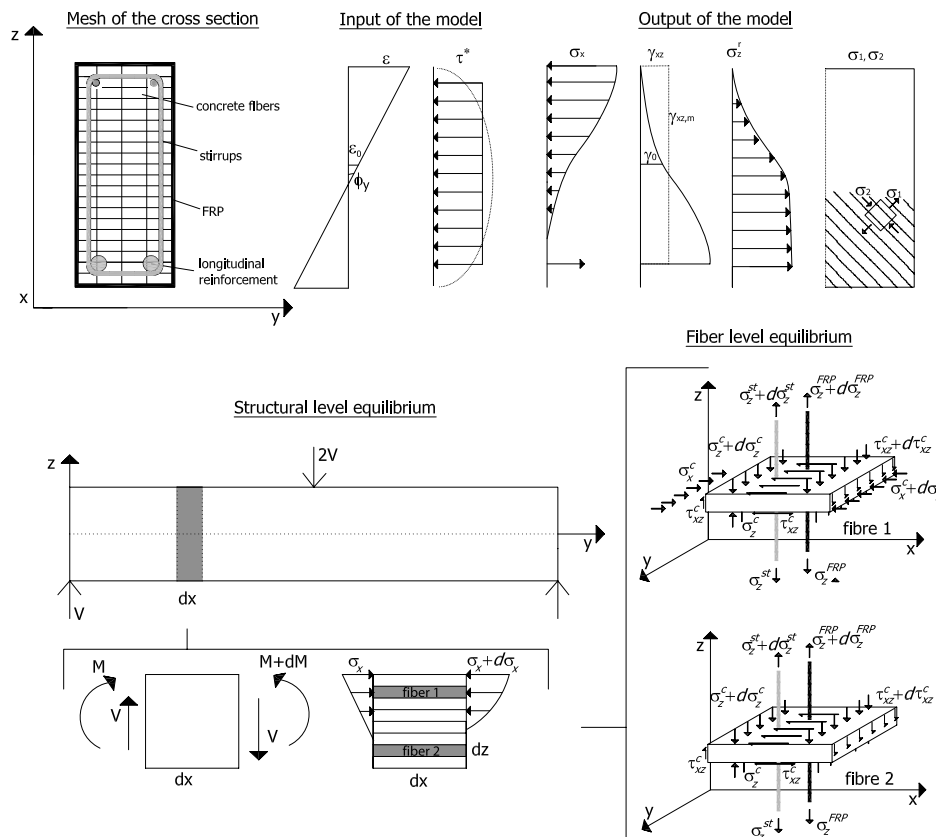


Fig. 3. Hybrid N-M-V sectional model

(2) the force assumption of constant shear stress along the cross section:

$$\tau^* = G^* A^* \gamma_0 \quad (3)$$

where G^* = transversal modulus; γ_0 = distortion at the neutral axis; and A^* = effective shear area, given by the summation of the areas of the shear resistant fibers. The fibers located in the concrete cover are considered to be under null shear stress. The transversal reinforcement constituted by steel stirrups and FRP is considered to be smeared in the concrete fibers.

The state determination of the fibers (constituted by concrete and smeared reinforcement and denoted as $c + st$) and the filaments (constituted by the longitudinal steel and denoted as sl) is performed in an incremental form and is summarily described in the following points. As a result, the stiffness matrix, K_{sec} , and the internal load vector, $S_{sec} = [NV_z M_y]^T$, of the cross section are given by the summation of both contributions as follows:

$$K_{sec} = K_{sec}^{c+st} + K_{sec}^{sl} \quad (4)$$

$$\underline{S}_{sec} = \underline{S}_{sec}^{c+st} + \underline{S}_{sec}^{sl} \quad (5)$$

Transversal reinforcements (steel and FRP) are considered to be smeared in the concrete fibers, meaning that their contributions to forces and stiffness are made through a ratio quantity and their stress-strain state is determined by means of independent constitutive laws. Even if it is discontinuous or applied in the external part of the cross section, the reinforcement is considered to be smeared in all of the shear resistant fibers. In the case of rectangular cross sections, the entire web, disregarding the cover part, is considered to be shear resistant. More complex geometries are provided in the proposals given in Ferreira et al. (2013).

By considering the N-M-V interaction, each fiber is under a 2D strain-stress state (Fig. 3). This strain-stress state can be written by the following equation, which relates the tensor of stresses with the tensor of strains by means of the stiffness matrix of the fiber D_{fiber} :

$$\begin{pmatrix} \Delta\sigma_x \\ \Delta\sigma_z \\ \Delta\tau_{xz} \end{pmatrix} = D_{fiber} \begin{pmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_z \\ \Delta\gamma_{xz} \end{pmatrix} \quad (6a)$$

$$D_{fiber} = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & \bar{D}_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix} \quad (6b)$$

The complete stiffness matrix of the fiber D_{fiber} [Eq. (6b)] is computed as the summation of both contributions (concrete and transversal reinforcement). A concrete fiber can have different n_k and n_l configurations of transversal steel and FRP sheets, which are accounted for in the model through their respective volumetric ratios, $\rho_{st,k}$ and $\rho_{FRP,l}$, and are submitted to axial stresses ($\sigma_{z,k}^{st}$, $\sigma_{z,l}^{FRP}$). Because the transversal reinforcement is only submitted to longitudinal stresses in the transversal direction of the section, its contribution is accounted for in the term D_{22} of the fiber stiffness matrix as follows:

$$\bar{D}_{22} = D_{22} + \rho_r E_r \quad (7a)$$

$$\rho_r E_r = \sum_{k=1}^{n_k} \left(\frac{A_{st,k}}{s_{st,k} b_{st,k}} E_{st,k} \right) + \sum_{l=1}^{n_l} \left(\frac{A_{FRP,l}}{s_{FRP,l} b_{FRP,l}} E_{FRP,l} \right) \quad (7b)$$

where $A_{st,k}$ = area of transversal steel; $b_{st,k}$ = width of the cross section; $s_{st,k}$ = longitudinal spacing of each configuration of stirrups k ; n_k = total number of configurations of stirrups in a cross section [that, in case of strengthened elements, can be greater than 1 (Ferreira et al. 2013)]; $A_{FRP,l}$ = area of transversal FRP strengthening; $b_{FRP,l}$ = width of the cross section; $s_{FRP,l}$ = longitudinal spacing of each configuration of FRP sheet l ; and n_l = total number of FRP sheet configurations in a cross section.

Compatibility requirements impose that the vertical strain, ε_z , in concrete is equal to the strain in the transversal reinforcement (steel stirrups and FRP strengthening).

The state of the fiber is determined by means of the following two requirements: (1) equilibrium in the transversal direction between the concrete and the vertical reinforcements (stirrups and FRP):

$$\Delta\sigma_z^c + \rho_r \Delta\sigma_z^r = 0 \quad (8a)$$

$$\rho_r \Delta\sigma_z^r = \sum_{k=1}^{n_k} \left(\frac{A_{st,k}}{s_{st,k} b_{st,k}} \Delta\sigma_{z,k}^{st} \right) + \sum_{l=1}^{n_l} \left(\frac{A_{FRP,l}}{s_{FRP,l} b_{FRP,l}} \Delta\sigma_{z,l}^{FRP} \right) \quad (8b)$$

and (2) the computed increment of shear stress, $\Delta\tau_{xz}$, must equate the imposed shear stress given by the fixed stress constraint $\Delta\tau^*$:

$$\Delta\tau^* - \Delta\tau_{xz} = 0 \quad (9)$$

This determination is not linear. An innermost iterative procedure within the fiber level is performed to compute the fiber stiffness matrix, K_{fiber} , and internal resistant force, S_{fiber} [more details are included in the study by Ferreira (2013)].

Once this iteration procedure reaches convergence, the state determination of the fiber is accomplished. Because stress, σ_z , is null and the section model does not include ε_z , a static condensation may be applied to Eq. (6), resulting in a condensed form of the stiffness matrix of the fiber K_{fiber} and the shear modulus G^* in Eq. (3) is given by the element $K_{fiber}(2,2)$:

$$K_{fiber} = \begin{pmatrix} D_{11} - \frac{D_{12}D_{21}}{D_{22}} & D_{13} - \frac{D_{12}D_{23}}{D_{22}} \\ D_{31} - \frac{D_{32}D_{21}}{D_{22}} & D_{33} - \frac{D_{32}D_{23}}{D_{22}} \end{pmatrix} \quad (10)$$

The contribution of the fibers to the sectional stiffness matrix K_{sec} and the internal force vector S_{sec} are computed through the integration of stiffness and forces of all of the fibers of the cross section, as follows:

$$K_{sec}^{c+st} = \int T^T K_{fiber} T dA \quad (11)$$

$$\underline{S}_{sec}^{c+st} = \int T^T \underline{S}_{fiber} dA \quad (12)$$

where A = area of each fiber; and T = Timoshenko transformation matrix that relates the generalized strains of the element to the strains in each fiber.

Longitudinal steel is considered under an axial strain-stress state. The contribution of the longitudinal reinforcement for the cross section can be computed as the integral of stiffness and stresses throughout all of the filaments:

$$K_{sec}^{sl} = \int T_{sl}^T E_{sl} T_{sl} dA_{sl} \quad (13)$$

$$\underline{S}_{sec}^{sl} = \int T_{sl}^T \sigma_x^{sl} dA_{sl} \quad (14)$$

where A_{sl} = area of each steel filament; and T_{sl} = transformation matrix related to the plane section theory.

Element Model

A two-node Timoshenko FE with linear shape functions was implemented in the model. Regarding the 2D case, the displacement field considers two displacements, axial and vertical, and one rotation. The classical equations for the element stiffness matrix, K_{elem} , and the internal resistant load vector, F_{elem} , are rewritten in the following manner, by making use of the formerly described sectional model:

$$K_{elem} = \int BK_{sec}Bdx \quad (15a)$$

$$F_{elem} = \int B^T S_{sec}dx \quad (15b)$$

These integrals are solved through the Gaussian quadrature method and reduced integration is considered to avoid shear locking (Zienkiewicz 1977).

Nonlinear Numerical Procedure

The presented nonlinear displacement based FE model is implemented into a Newton-Raphson framework to solve the global equations of equilibrium. The response of a frame structure along the different stages of damage—elastic, cracked, yielded and ultimate levels—is reproduced by the model. Also, the nonlinear effects of shear and its interaction with normal forces are considered within all of the ranges of damage.

Because the proposed model for strengthened structures is based on a fiber beam 1D formulation with a simplified approach to account for N-M-V interaction, it allows the nonlinear analysis

of FRP strengthened RC elements in an accurate yet simple manner. The model is capable of predicting shear failures, taking into account the contributions of concrete, stirrups, and FRP reinforcements to the resistance mechanism of the beam. It also predicts the load correspondent to the start of loading for the transversal reinforcements, associated with the start of diagonal cracking and ending of the concrete contribution to the tensile ties of the shear resistant mechanism. The nonlinear development of deflections with loading, stresses, and strains in concrete and reinforcement, principal direction of the stress flows, and cracking patterns are also computed by the 1D model.

Comparison with Experimental Results

The model described in the previous section is compared with an experimental campaign of RC beams strengthened in shear with externally bonded FRP sheets. Experimental data on deflections and strains measured in the steel stirrups and in the FRP reinforcement are compared with the numerical predictions. After this, numerical predictions related with shear resistance mechanism are presented and discussed, such as the development of stresses and strains in the stirrups and in the sheets, the inclination of the angles of the compression struts, and the flow of transversal strains in the shear critical cross section.

Description

An experimental campaign on shear strengthened RC beams was conducted by Alzate (2012) with the purpose of studying the contribution of FRP to the shear resistance of RC elements. The beams were simply supported, 4.5 m long, and with a rectangular cross section of 0.42 m height and 0.25 m width. Along with the control specimen, which corresponds to the beam without strengthening, two shear-strengthened beams with different FRP wrapped quantities were simulated (W90S5 and W90S3). The geometry,

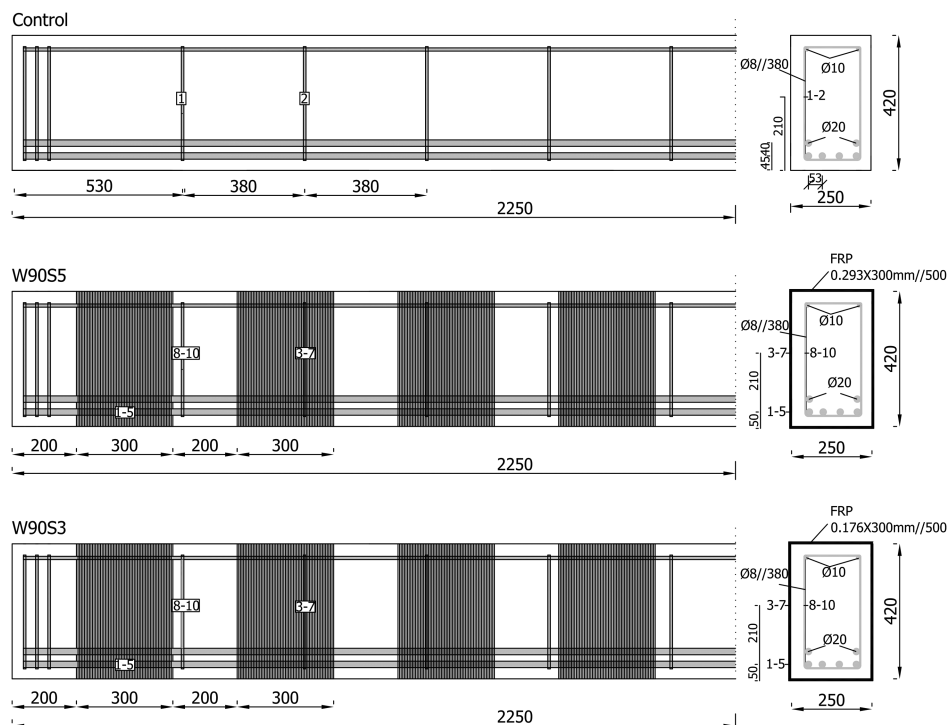


Fig. 4. Geometry, reinforcement, strengthening configurations, and instrumentation of the beams tested by Alzate (2012) (dimensions in mm)

reinforcement, and strengthening configurations are presented in Fig. 4. The notation indicates that the fibers of the FRP sheets formed an angle of 90° with respect to the longitudinal axis; the sheets were spaced (S) at 200 mm from edge to edge and were made of two types of unidirectional fibers (Types 5 or 3; 530 or 300 g/m²) with two different thicknesses (0.293 and 0.165 mm for Types 5 and 3, respectively). The corners of the section were properly prepared to bond the FRP sheets. The bottom corners had a minimum radius of 25 mm, made through the use of a concave molding in the formwork. The top corners were rounded by polishing the surface.

Regarding the characterization of materials, only the cylinder compressive strength of concrete was experimentally determined (36.95, 24.47, and 22.64 MPa for the control beam, W90S5, and W90S3, respectively); these values were considered in the model. The reinforcement steel belongs to the class with yielding strength of 500 MPa and the following values were considered in the model ($f_{sy} = 550$ MPa, $E_s = 200$ GPa, $\varepsilon_{sy} = 2.75 \times 10^{-3}$, and $\varepsilon_{su} = 0.12$). The mechanical properties used to simulate FRP were given by the provider [elastic modulus, $E_f = 240$ GPa for both fiber types; ultimate strength, $f_{fu} = 4,000$ MPa for Type 5; and 3,800 MPa for Type 3 (Alzate 2012)]. Further material properties considered in the model are listed in Table 1.

Each beam was submitted to two load tests with different total spans and equal shear spans, as presented in Fig. 5: the configuration of the first test includes the entire span of the beam and

the concentrated load is applied at a distance of three times the total depth ($a = 3h = 1.26$ m) from the support (entitled the long-span test); after the first failure in one end, a second test of the undamaged part of the beam is performed, considering a total span of 3.04 m and locating the concentrated load at the opposite side at a distance of 3 h (1.26 m) from the support. For the case of W90S3, two beams were tested (W90S3a and W90S3b). Pertaining to the instrumentation applied in each beam, the vertical displacements at midspan were measured by means of displacements transducers and the vertical strains in the stirrups and the FRP were monitored at the shear span by means of bonded strain gages. The locations of the sensors are presented in Fig. 4.

Concerning the numerical model, the FE mesh is presented in Fig. 5. The beams were simulated by means of beam elements with 0.1 m in length, the cross section was discretized into fibers of 0.005 m in height, the longitudinal reinforcement was simulated with steel filaments, and both the transversal steel and FRP reinforcement were assumed to be smeared in the concrete fibers. Load was applied incrementally until failure.

Results and Discussion

The shear resistance predicted by the numerical model, $V_{u,num}$, is compared with the experimental results, $V_{u,exp}$, in Table 1. In general, a very good fitting is achieved, with an average of $V_{u,num}/V_{u,exp}$ of 1.07. The higher difference between these two

Table 1. Results at Failure: Different Contributions to Shear Resistance

		Material properties (according to Fig. 2)			Total shear resistance (kN)			Nonstrengthened shear resistance (kN)			FRP contribution to shear resistance (kN)				
Beam	Test	Concrete	Steel	FRP	$V_{u,exp}$	$V_{u,num}$	$V_{u,num}/V_{u,exp}$	$V_{1,exp}$	$V_{1,num}$	$V_{1,num}/V_{1,exp}$	$V_{f,exp}$	$V_{f,num}$	$V_{f,num}/V_{f,exp}$		
Control	Long	$f_c = 36.95$ MPa	$f_{sy} = 550$ MPa	—	149	154	1.03	149	154	1.03	—	—	—		
	Short	$E_c = 32.56$ GPa	$f_{su} = 580$ MPa		158	173	1.09	158	173	1.09	—	—	—		
		$\varepsilon_0 = 0.00227$	$E_s = 200$ GPa												
		$\varepsilon_u = 0.0035$	$\varepsilon_{sy} = 2.75 \times 10^{-3}$												
		$f_{ct} = 3.33$ MPa	$\varepsilon_{su} = 0.12$												
		$\varepsilon_{cr} = 0.0035$													
		$\varepsilon_{ct,u} = 0.002$													
W90S5	Long	$f_c = 24.47$ MPa	$f_{sy} = 550$ MPa	$f_{fu} = 4,000$ MPa	276	281	1.02	135	136	1.01	141	145	1.03		
	Short	$E_c = 28.77$ GPa	$f_{su} = 580$ MPa	$E_f = 240$ GPa	374	296	0.79	143	153	1.07	231	143	0.62		
		$\varepsilon_0 = 0.0017$	$E_s = 200$ GPa	$\varepsilon_{f,u} = 0.015$											
		$\varepsilon_u = 0.0035$	$\varepsilon_{sy} = 2.75 \times 10^{-3}$												
		$f_{c,c} = 46.9$ MPa	$\varepsilon_{su} = 0.12$												
		$E_{c,c} = 34.98$ GPa													
		$\varepsilon_{0,c} = 0.0027$													
		$\varepsilon_{u,c} = 0.018$													
		$f_{ct} = 3.90$ MPa													
		$\varepsilon_{cr} = 0.000112$													
		$\varepsilon_{ct,u} = 0.002$													
W90S3	Long (a)	$f_c = 22.64$ MPa	$f_{sy} = 550$ MPa	$f_{fu} = 3,800$ MPa	311	277	0.89	132	132	1.00	179	145	0.81		
	Long (b)	$E_c = 28.11$ GPa	$f_{su} = 580$ MPa	$E_f = 240$ GPa	284	277	0.98	141	132	0.94	152	145	0.95		
	Short (a)	$\varepsilon_0 = 0.0016$	$E_s = 200$ GPa	$\varepsilon_{f,u} = 0.0150$	321	297	0.93	133	150	1.13	181	147	0.81		
	Short (b)	$\varepsilon_u = 0.0035$	$\varepsilon_{sy} = 2.75 \times 10^{-3}$		277	297	1.07	141	150	1.06	136	147	1.08		
		$f_{c,c} = 37.0$ MPa	$\varepsilon_{su} = 0.12$												
		$E_{c,c} = 32.57$ GPa													
		$\varepsilon_{0,c} = 0.00227$													
		$\varepsilon_{u,c} = 0.013$													
		$f_{ct} = 3.33$ MPa													
		$\varepsilon_{cr} = 0.000102$													
		$\varepsilon_{ct,u} = 0.002$													
		Average							0.98			1.04			0.88
		SD							0.10			0.06			0.17

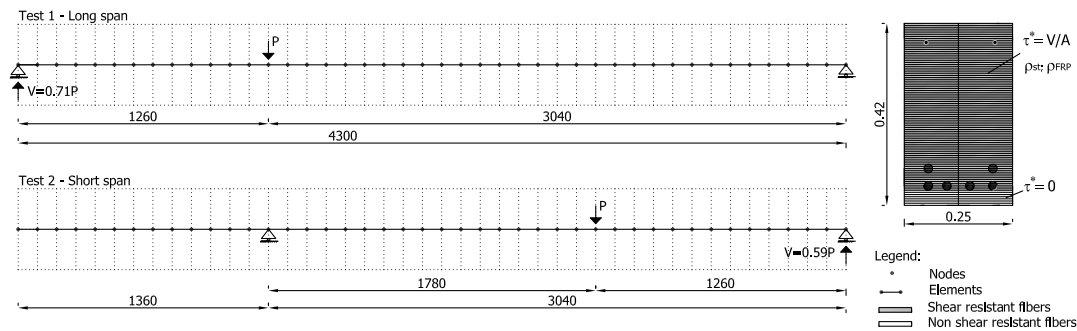


Fig. 5. Test setup and mesh of the numerical model (dimensions in mm)

values is related to the short span test of Beam W90S5 that presented a nonstandard shear resistance, significantly higher than all the other tests; this was also mentioned by the authors of the experimental campaign (Alzate 2012). The different contributions to shear resistance of the strengthened beams are shown in Table 1.

- V_1 represents the shear strength of the beam without FRP: the experimental value $V_{1,exp}$ is given by Alzate (2012) and has been determined by an extrapolation, proportional to the cubic root of the ratio between the concrete compressive strengths of the tested and control beams (Alzate 2012); the theoretical shear strength, $V_{1,num}$, is determined by the numerical model by simulating Beams W90S5 and W90S3 and disregarding the FRP strengthening.
- V_f represents the shear conducted by the FRP and is determined by subtracting the shear strength of the beams without strengthening (V_1) to the ultimate shear capacity of the

beams (V_u), in both the experimental ($V_{f,exp}$) and numerical cases ($V_{f,num}$).

The efficiency of the model to simulate the load capacity of RC shear critical beams has already been demonstrated in the works of Ferreira (2013) and Ferreira et al. (2013). However, this capacity is confirmed herein by the correct prediction of the shear capacity of unstrengthened beams. Moreover, considering the complexity involved in the shear mechanism of strengthened elements, the most noteworthy aspect of the numerical model is its capacity to account for the contribution of the FRP sheets to the shear resistance of the beams. This was demonstrated by the fitting achieved between the experimental $V_{f,exp}$ and predicted $V_{f,num}$ values.

Failure is predicted to be shear related, with the stirrups achieving ultimate limit strain for the case of the control beams and shear bending related with failure of FRP with crushing of concrete for the case of the strengthened beams. Pertaining to the experimental

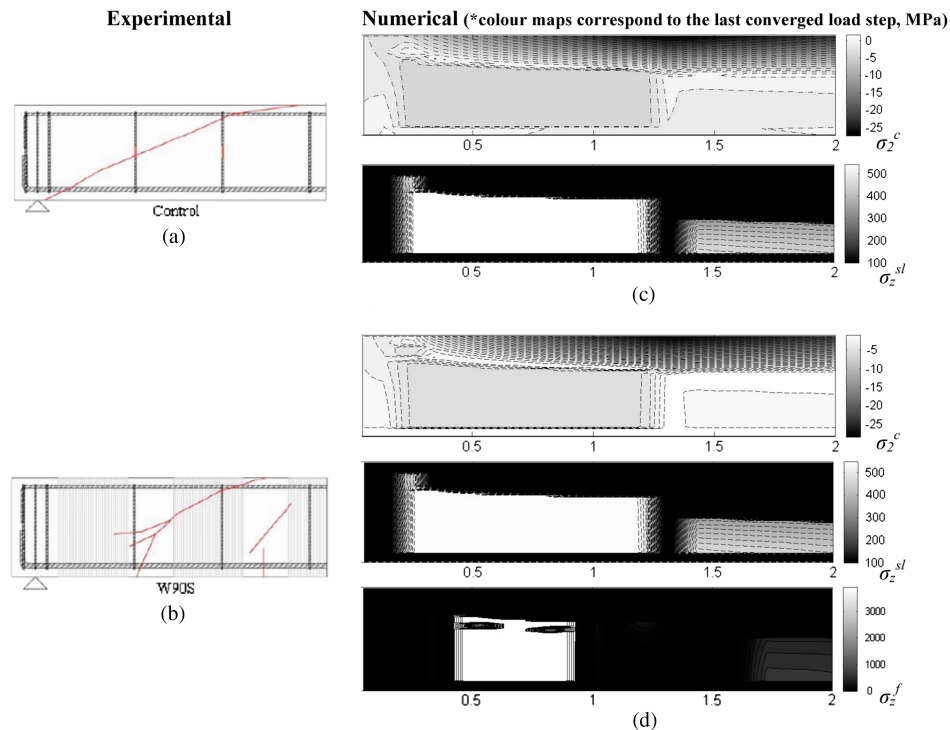


Fig. 6. Experimental and numerical damage at failure for the control and strengthened beams: (a) experimental shear failure of the control beam with main diagonal crack; (b) experimental bending-shear failure of the strengthened beam with failure of FRP and crushing of concrete at load application point; (c) numerical shear failure of the control beam with failure of stirrups; (d) numerical bending-shear failure with failure of FRP, crushing of concrete, and yielding of longitudinal and transversal reinforcement

observations given by Alzate (2012), shear failures were for the control specimens. For the FRP strengthened beams, two types of failures were observed: one accompanied by FRP rupture in the corners, and in the other, concrete crushing was observed near the load application point (Alzate 2012). Fig. 6 shows a summary of the computed and experimentally observed damage at failure stage. Pertaining to the experimental tests, the typical cracking patterns at failure are represented. In relation to the numerical results, the maps of the distribution of stresses in stirrups and FRP and the principal compression stresses in concrete are represented along the beams and are related to the last converged load step before reaching failure. These maps demonstrate the existence of failure for the formerly cited reasons: the control beam presents stirrups yielding in almost all the shear span zone; FRP strengthened beams present FRP tensile rupture in the center shear span zone, together with concrete crushing near the load point. Also, yielding of longitudinal reinforcement was predicted in the strengthened beams.

Because the N-V-M fiber beam model computes the nonlinear response of the frame elements along various levels of loading, the displacements and strains in the stirrups and in the FRP monitored in the experimental campaign were compared with the predicted results. Some of these comparisons are presented in the following.

The computed load-displacement curves at midspan are depicted, along with the experimental results, in Fig. 7 for the control and strengthened beams (W90S3a, W90S3b, and W90S5). All results are related to the long-span tests. In fact, the short-span tests presented some unforeseen responses. Because this is the second test performed in each specimen, there is doubt whether the previous damage induced by the first test might affect the response of the subsequent short-span test. For these reasons, the results related to the short-span tests are not included in this section. The increment of load capacity of the strengthened beams in relation to the control specimen is very perceptible in these graphics because it is well captured by the model. Hence, in all these cases, as shown in Fig. 7, the numerical model predicted the overall experimental response of the beams with satisfactory accuracy, both in terms of the ultimate load-carrying capacity and the nonlinear development of displacements with increasing load up to failure.

Fig. 8 shows the experimental results of strains in the stirrups measured by Sensors 8–10 in the shear span of the strengthened beams (as shown in position in Fig. 4), along with the computed predictions. Concerning the response of the stirrups, the model is able to predict, with relative accuracy, the start of loading of

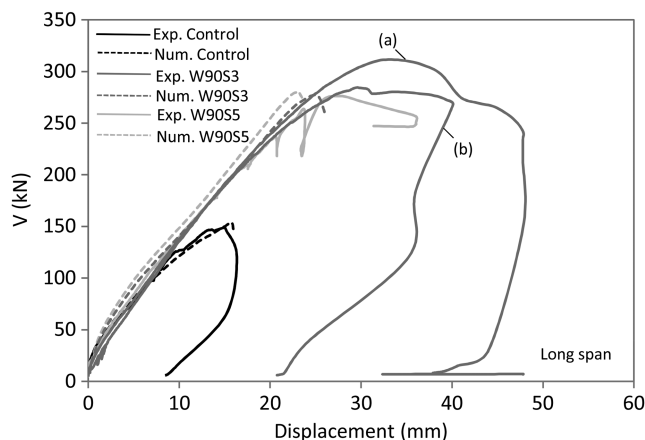


Fig. 7. Shear versus midspan deflection for the control and strengthened long span beams: numerical and experimental results

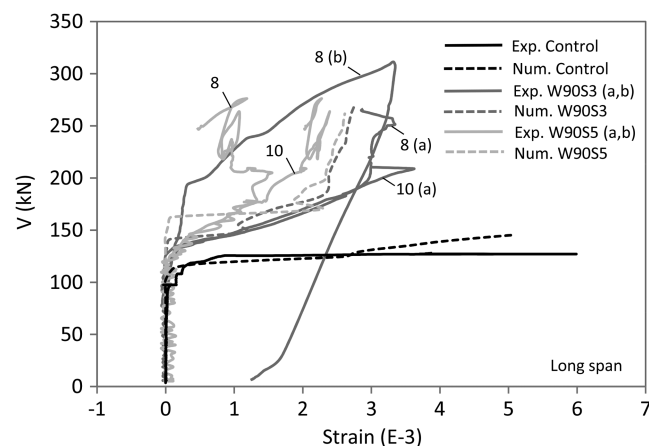


Fig. 8. Strains in stirrups in the control (Sensor 1) and in the strengthened (Sensors 8, 10) long-span beams: numerical and experimental results

the transversal steel and the overall tendency of the development of strains.

By comparing the control and strengthened beams, it is perceived that the loading level for which the stirrups start to develop significant stress (that is coincident with the end of the so-called concrete contribution to shear resistance) starts slightly later in the presence of FRP strengthening. The concrete contribution to the shear resistance, V_{cu} , increases with the FRP reinforcement ratio, primarily because of the concrete confinement effect produced by the FRP in a wrapped configuration. Also, in the shear strengthened beams, the stirrups are maintained in the elastic stage for higher levels of loading, reaching yielding at a more advanced stage than the control beam because of the shear contribution of FRP.

Subsequently, the strains measured in the FRP sheets are compared with the numerical predictions. The results corresponding to Sensors 3–7 for strengthened beams are presented in Fig. 9 (Fig. 4 shows the locations of the sensors). A later activation of the FRP sheet in relation to the stirrups in the approximate same position is observed and reported (Alzate 2012); however, the model is not able to reproduce this difference. In fact, because the model considers a perfect bond between the concrete and the reinforcements

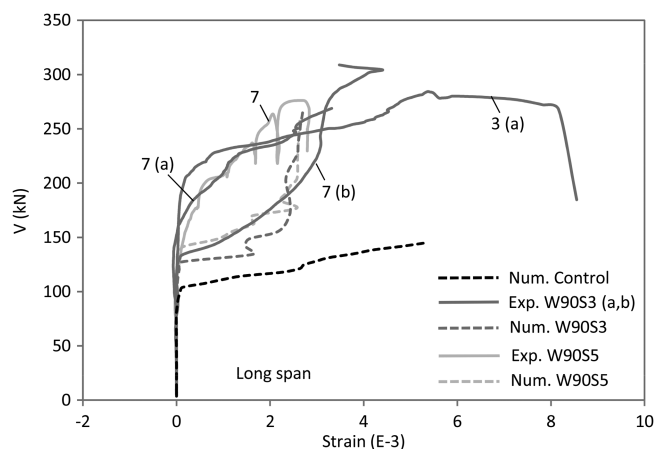


Fig. 9. Strains in the transversal reinforcement, FRP sheets for the strengthened long span beams, and stirrups in the control long-span beam: numerical and experimental results

(steel and FRP), it does not account for possible bond-slip actions. Nevertheless, because of the inherent difficulty in simulating the FRP response in the shear response of RC beams, the computed results are considered to reasonably address the experimental behavior.

According to the numerical predictions of Figs. 8 and 9, the transversal reinforcement starts to carry load at almost 100 kN for the case of the control beams, at 120 kN for the beam with less quantity of FRP (W90S3), and at 134 kN for the beam with higher quantity of FRP (W90S5).

Additional Numerical Predictions

This section provides in-depth analyses of other aspects related to the resisting mechanisms of the modeled beams (control beam, W90S3, and W90S5). The results relate to the modeling of the long span tests.

Although the strains in the stirrups and FRP are considered to be equal in the numerical model, their correspondent stresses and contribution to the shear load carrying capacity of the beams are different. Hence, the computed stresses in the transversal reinforcement σ_z^r (stirrups and FRP) with increasing load are depicted in Fig. 10. For the case of the strengthened beams, the results of the stirrups correspond to Positions 9, 11 and those of the FRP to Positions 3, 7; for the control beam, the stresses in the stirrups correspond to Position 2 (Fig. 4). Before yielding of the steel, the stirrups and FRP have similar responses; after yielding of the stirrups, the stresses of FRP increase dramatically until reaching the ultimate strength. Comparably, in the control beam, the stresses in the stirrups develop for lower load stages, and this difference is more accentuated than the beam with a higher ratio of FRP (W90S5).

The distributions of vertical strains, ϵ_z , along the cross section located at the midshear span of the beams are depicted in Fig. 11. Two load levels are presented: $V = 130$ kN, which corresponds to an advanced damage state of the control beam and a low damage state of the strengthened beams, and $V = 250$ kN, corresponding to a state near failure in the beams with FRP shear strengthening (in the former case, the control beam cannot be represented). The proposed model predicts a triangular shaped distribution of strains (higher strains in the cracked area and almost null strains in the uncracked compression top area), which accurately approximates the predictions from more complex section models of the interaction of forces (Bairán and Mari 2007a; Mohr et al. 2010), as demonstrated in the work of Ferreira (2013). The effects of FRP on lowering the vertical stresses is shown in Fig. 11, because for

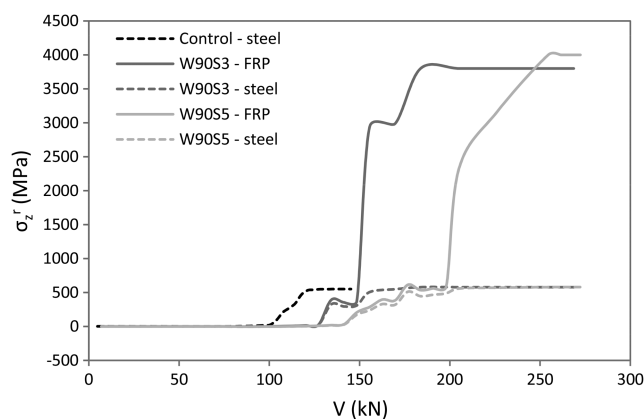


Fig. 10. Numerical stresses in the transversal reinforcement versus shear force for the long-span beams

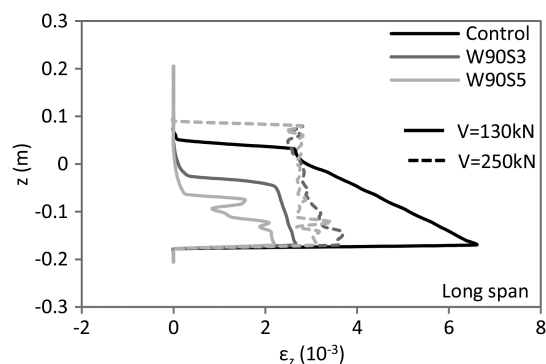


Fig. 11. Numerical vertical strains at the section $x = 0.63$ m for the long span beams

the same load level, the increase of FRP diminishes the strains in the section. In the state near failure, the vertical strains in the strengthened beams present similar responses. In addition, the presence of FRP diminishes the shear strains in the cross section.

Fig. 12 shows the development of stresses in stirrups and FRP along the shear critical cross section of Beam W90S3 for different load levels. The FRP is significantly activated only for load levels corresponding to advanced damage and after yielding of the transversal steel. The stresses in tensile longitudinal reinforcement at the sections of midshear span and under load are presented in Fig. 13

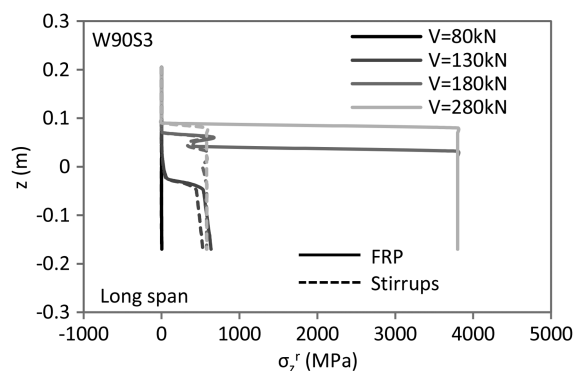


Fig. 12. Numerical stresses in the transversal reinforcement at the section $x = 0.63$ m of the long-span beam W90S3

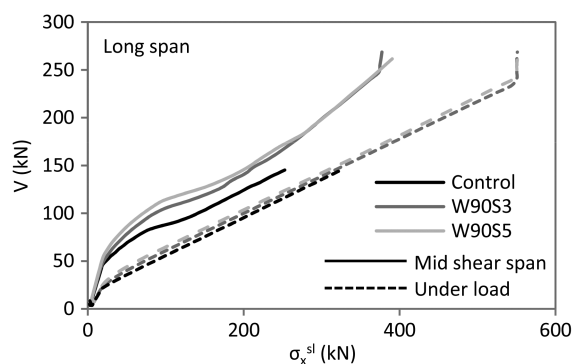


Fig. 13. Numerical stresses in the longitudinal reinforcement versus shear force at the section $x = 0.63$ m (midshear span) and $x = 1.26$ m (under load) for the long-span beams

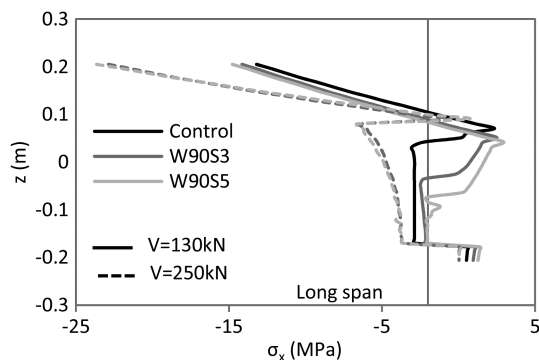


Fig. 14. Numerical longitudinal stresses at the section $x = 0.63$ m for the long-span beams

for the beams with different ratios of FRP shear strengthening. The presence of FRP delays the development of these stresses in the presence of shear stresses (in the midshear span section). In the bending critical section under the application load point, the presence of FRP does not influence these results because similar curves were predicted.

The distributions of computed stresses in concrete along the same cross section are also presented in Fig. 14 for two load levels. Because of the consideration of the nonlinear shear bending resistant mechanism in the numerical model, compressive stresses arise in the cracked area below the neutral axis. These stresses correspond to the longitudinal component of the diagonal compressive struts and are in good agreement with more complex models (Bairán and Marí 2007a; Mohr et al. 2010), as described in the works of Ferreira (2013) and Ferreira et al. (2013). Because the equilibrium of forces and moments in the cross section has to be fulfilled, these compressive stresses in the tensile zone of the cross section must be balanced through the increment of tensile stresses in the longitudinal reinforcement. This represents the so-called tension-shift effect of the longitudinal reinforcement in the presence of shear forces. The presence of FRP lowers the longitudinal component of the diagonal compression struts (as in Fig. 14 for $V = 130$ kN), which is directly related to the diminishing of the tensile-shift effect of the longitudinal reinforcement shown for the shear critical section at midshear span in Fig. 13.

The average principal compression angle at the midshear span with shear force is presented in Fig. 15 for beams with different ratios of FRP. The average angle of the compression struts ranges from 40° to nearly 25° – 20° with increasing load. The presence of

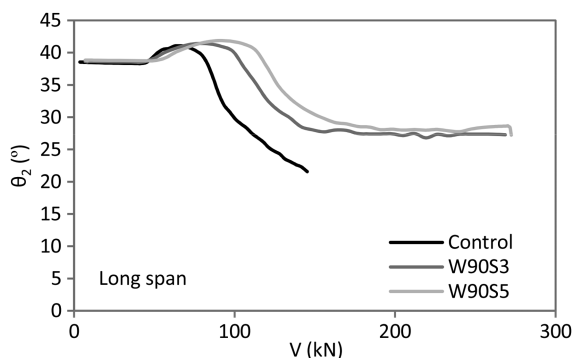


Fig. 15. Numerical average principal compression angle versus shear force at the section $x = 0.63$ m for the long-span beams

FRP delays this rotation, i.e., for the same load level, the higher ratios of FRP originate less inclined compression struts (increasing the average principal compression angle).

Conclusions

A fiber beam model for the nonlinear analysis of RC frames subjected to combined normal and shear forces, previously developed by the authors, has been extended to include the response of FRP shear strengthened elements. The FRP extended model has been compared with eight experimental results available in literature for RC beams strengthened in shear in fully wrapped CFRP sheets. Four experimental beams were analyzed (control, W90S5, W90S3a, and W90S3b) in which each beam was submitted to two load tests (long span and short span). Long-span tests were compared in terms of ultimate load, displacement, strains in stirrups and FRP. Short-span tests were compared in terms of maximum load at failure and the contribution of the different materials. The model reproduces, with reasonable accuracy, the experimental failure loads and the load-deflection behavior; moreover, it has been used to study other aspects including the angle of strut inclination and the evolution of vertical strain along the shear span. The aspects described in the following have been represented by the model.

- The loading level for which the stirrups start to develop significant stress (coincident with the initiation of diagonal cracking and with the end of the so-called concrete contribution to shear resistance) starts slightly later in the presence of FRP strengthening. The concrete contribution to the shear resistance increases as the amount of strengthening FRP increases. This is primarily because of the concrete confinement provided by the FRP in a wrapped configuration. Increments of compressive strength as a result of FRP confinement were considered by the expression of Spoelstra and Monti (1999).
- In the FRP shear strengthened beams under study, the stirrups are maintained in the elastic stage for higher levels of loading, reaching yielding at a more advanced stage than those without FRP strengthening. This is because of the contribution of FRP to carry the shear forces applied to the beam.
- Before yielding of the steel, stirrups and FRP have similar responses; after yielding of the stirrups, FRP dramatically increases its stresses until reaching the ultimate strength.
- For lower load levels, vertical strains at the stirrups and FRP are almost null; these strains increase after the onset of cracking and continue rising as cracking develops until failure.
- The presence of FRP lowers the longitudinal component of the diagonal compression struts, which is directly related to the diminishing of the tensile-shift effect of the longitudinal reinforcement.
- The average angle of the compression struts ranges from 40° to nearly 25° – 20° with increasing load. The presence of FRP delays this rotation, i.e., for the same load level, the higher ratios of FRP originate less inclined compression struts.
- The failure mechanism of the strengthened beams was not purely attributable to shear; it involved FRP failure and local concrete crushing in the loading point. Accordingly, the numerical model detected concrete crushing in the load application point, together with FRP tensile failure in the midshear span. According to the numerical results, it appears that the presence of FRP reinforcement modifies the shear resistant mechanism (inclinations to cracks and struts and concrete confinement stresses), affecting the concrete and internal steel components of shear strength. Thus, an interaction between the three shear resisting components has been detected, which requires further investigation

to formulate a design approach that reduces the scatter in the analytical predictions with respect to the experimental results.

Furthermore, the described model will be extended to simulate the behavior of other FRP shear configurations such as U-shaped and side bonded sheets through the implementation of a debonding criterion and a bond-slip relationship between the sheets and the support.

Acknowledgments

This work has been developed in the ambit of the research projects "Performance-based-design of partially prestressed concrete structures. Proposal of new design methodology, experimental verification and design criteria" (BIA2012-36848), financed by the Spanish Ministry of Economics and Competitiveness; and "Rehabilitation of roads and motorways" (REHABCAR), funded by the Spanish Ministry of Science and Innovation. Funding provided by the Portuguese Foundation for Science and Technology (FCT) to the first author through the Ph.D. grant SFRH/BD/43232/2008 is also gratefully acknowledged. The authors acknowledge the support of Albert Alzate, Angel Arteaga, Daniel Cisneros and Ana de Diego from the Instituto de Ciencias de la Construcción Eduardo Torroja of Spain, on the provided data related to their experimental program.

References

- Al-Sulaimani, G. J., Sharif, A. M., Basunbul, I. A., Baluch, M. H., and Ghaleb, B. N. (1994). "Shear repair for reinforced concrete by fiberglass plate bonding." *ACI Struct. J.*, 91(4), 458–464.
- Alzate, A. (2012). "Análisis de los modelos de comportamiento de vigas de hormigón armado reforzadas a cortante con polímeros armados con fibras (FRP). Validación y calibración experimental." Ph.D. thesis, Madrid, Universidad Politécnica de Madrid (in Spanish).
- Arduini, M., Nanni, A., Di Tommaso, A., and Focacci, F. (1997). "Shear response of continuous RC beams strengthened with carbon FRP sheets." *Proc., 3rd Int. Symp. on Non-metallic (FRP) Reinforcement for Concrete Structures*, Vol. 1, Japan Concrete Institute, Tokyo, 459–466.
- Bairán, J., and Marí, A. (2007a). "Multiaxial-coupled analysis of RC cross-sections subjected to combined forces." *Eng. Struct.*, 29(8), 1722–1738.
- Bairán, J., and Marí, A. (2007b). "Shear-bending-torsion interaction in structural concrete members: a nonlinear coupled sectional approach." *Arch. Comput. Methods Eng.*, 14(3), 249–278.
- Bazant, Z. (1983). "Comment on orthotropic models for concrete and geomaterials." *J. Eng. Mech.*, 109(3), 849–865.
- Bousselham, A., and Chaallal, O. (2006). "Behavior of RC T-beams strengthened in shear with CFRP: An experimental study." *ACI Struct. J.*, 103(3), 339–347.
- Bousselham, A., and Chaallal, O. (2008). "Mechanisms of shear resistance of concrete beams strengthened in shear with externally bonded FRP." *J. Compos. Constr.*, 10.1061/(ASCE)1090-0268(2008)12:5(499), 499–512.
- Caiazza, R., Recupero, A., and Scilipoti, C. D. (2006). "Collasso di travi in C.A. per taglio e utilizzo di FRCM per l'adeguamento." *Proc., Convegno Nazionale Crolli e Affidabilità delle strutture civili (CRASC'06)*, Università degli Studi di Messina, Italy (in Italian).
- Carol, A., and Taljsten, B. (2005). "Experimental study of strengthening for increased shear bearing capacity." *J. Compos. Constr.*, 10.1061/(ASCE)1090-0268(2005)9:6(488), 488–496.
- Ceresa, P., Petrini, L., Pinho, R., and Sousa, R. (2009). "A fibre flexure-shear model for seismic analysis of RC-framed structures." *Earthquake Eng. Struct. Dyn.*, 38(5), 565–586.
- Cervenka, V. (1985). "Constitutive model for cracked reinforced concrete." *ACI J.*, 82(6), 877–882.
- Chajes, M. J., Januszka, T. F., Mertz, D. R., Thomson, T. A. Jr., and Finch, W. W., Jr. (1995). "Shear strengthening of reinforced concrete beams using externally applied composite fabrics." *ACI Struct. J.*, 92(3), 295–303.
- Czaderski, C., and Motavalli, M. (2004). "Fatigue behavior of CFRP L-shaped plates for shear strengthening of RC T-beams." *Compos. Part B*, 35(4), 279–290.
- Deniaud, C., and Cheng, J. J. R. (2004). "Simplified shear design method for concrete beams strengthened with fiber reinforced polymer sheets." *J. Compos. Constr.*, 10.1061/(ASCE)1090-0268(2004)8:5(425), 425–433.
- Ferreira, D. (2013). "A model for the nonlinear, time-dependent and strengthening analysis of shear critical frame concrete structures." Ph.D. thesis, Dept. of Construction Engineering, Universitat Politècnica de Catalunya, Barcelona, Spain.
- Ferreira, D., Bairán, J. M., and Marí, A. (2013). "Numerical simulation of shear-strengthened RC beams." *Eng. Struct.*, 46, 359–374.
- Godat, A., Neale, K. W., and Labossière, P. (2007). "Numerical modeling of FRP shear-strengthened reinforced concrete beams." *J. Compos. Constr.*, 10.1061/(ASCE)1090-0268(2007)11:6(640), 640–649.
- Khalifa, A., and Nanni, A. (2000). "Improving shear capacity of existing RC T-section beams using CFRP composites." *Cem. Concr. Compos.*, 22(3), 165–174.
- Lee, H. K., Kim, B. R., and Ha, S. K. (2008). "Numerical evaluation of shear strengthening performance of CFRP sheets/strips and sprayed epoxy coating repair systems." *Compos. Part B*, 39(5), 851–862.
- Malek, A., and Saadatmanesh, H. (1998). "Ultimate shear capacity of reinforced concrete beams strengthened with web-bonded fiber reinforced plastic plates." *ACI Struct. J.*, 95(4), 391–399.
- Marí, A. R. (2000). "Numerical simulation of the segmental construction of three dimensional concrete frames." *Eng. Struct.*, 22(6), 585–596.
- Mohr, S., Bairán, J., and Marí, A. R. (2010). "A frame element model for the analysis of reinforced concrete structures under shear and bending." *Eng. Struct.*, 32(12), 3936–3954.
- Petrangeli, M., Pinto, P. E., and Ciampi, V. (1999). "Fiber element for cyclic bending and shear of RC structures. I: Theory." *J. Eng. Mech.*, 125(9), 994–1009.
- Spolstra, M. R., and Monti, G. (1999). "FRP-confined concrete model." *J. Compos. Constr.*, 10.1061/(ASCE)1090-0268(1999)3:3(143), 143–150.
- Teng, J. G., Chen, J. F., Smith, S. T., and Lam, L. (2002). *FRP strengthened RC structures*, Wiley, New York.
- Triantafyllou, T. C. (1998). "Shear strengthening of reinforced concrete beams using epoxy-bonded FRP composites." *ACI Struct. J.*, 95(2), 107–115.
- Vecchio, F. J., and Bucci, F. (1999). "Analysis of repaired reinforced concrete structures." *J. Struct. Eng.*, 10.1061/(ASCE)0733-9445(1999)125:6(644), 644–652.
- Vecchio, F. J., and Collins, M. P. (1988). "Predicting the response of reinforced concrete beams subjected to shear using the modified compression field theory." *ACI J.*, 85(3), 258–268.
- Wong, R., and Vecchio, F. J. (2003). "Towards modelling of reinforced concrete members with externally bonded fiber-reinforced polymer (FRP) composites." *ACI Struct. J.*, 100(1), 47–55.
- Zienkiewicz, O. C. (1977). *The finite element method*, McGraw-Hill, London.